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An Extended Model for the Description of Phase Transitions in Shape Memory Alloys

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Certain materials like NiTi, CuZn, CuAuNi show strong memory effects when heated after having applied a deformation force. This behaviour is caused by an austenite-martensite phase transition in the crystal lattice which depends strongly on temperature. A mathematical theory for this kind of phase transitions has been set up in the past ten years, starting with the more physically oriented papers by M. Aehenbach and I. Muller [1982] and continued by more rigorous treatment of H. W. Alt, M. Niezgodka [1984] and M. Niezgodka, J. Sprekels [1985]. These papers make use of Landau's theory of phase transitions by introducing quite a complicated nonconvex form of the free-energy potential function. These investigations are limited to one space dimension. On the other hand the mathematical theory for this model is already quite good developed.

In our lecture we will use a model, which was developed recently by M. Fremond [1988]. We will give some mathematical contributions to the austenite-martensite phase transitions in the multidimensional situation. Our theory makes use of convex nonsmooth analysis. Based on the equations for energy balance

$$\dot{e} + \operatorname{div} q = \sigma\dot{\varepsilon} + \vec{M}\,\operatorname{grad}(\operatorname{tr}\dot{\varepsilon})$$

(e, q, σ, $\vec{M}$, ε stand for internal energy, heat flux, stress, internal force, strain, respectively) and for equilibrium in the quasi-static-case

$$\operatorname{div}\{\rho\nu\cdot\Delta[(\operatorname{div}\vec{u})I_3 + \sigma]\} = 0$$

(ρ, ν, $\vec{u}$, I_3 stand for density, curvature, displacement, 3×3 unit matrix, respectively), we obtain after some transformations and after introducing appropriate constitutive relations the following dynamical system:

$$(1)\qquad c_0\dot{\theta} - \sqrt{2}\dot{X}_1\alpha'(\theta)X_2\operatorname{div}\vec{u}_t - \sqrt{2}(\theta\alpha'(\theta)-\alpha(\theta))\operatorname{div}\vec{u}\,\dot{X}_2 = k\Delta\theta$$

$$(2)\qquad \operatorname{div}\{-\nu\Delta(\operatorname{div}\vec{u})I_3 - \lambda(\operatorname{div}\vec{u})I_3 + 2\mu\varepsilon(\vec{u}) - \sqrt{2}\alpha(\theta)X_2I_3\} = 0$$

$$(3)\qquad \kappa\begin{pmatrix}\dot{X}_1\\ \dot{X}_2\end{pmatrix} - \begin{pmatrix}\Delta X_1\\ \Delta X_2\end{pmatrix} + \partial I_K(X_1,X_2) \ni \begin{pmatrix}-(\sqrt{2}/\theta_0)(\theta-\theta_0)\\ -2\alpha(\theta)\operatorname{div}\vec{u}\end{pmatrix}$$

The system (1)-(3) is complemented by initial and boundary conditions. In this system X_i stand for the percentage of corresponding transformed material, α stands for the thermal extension coefficient and θ_0 for critical transformation temperature, c_0, k, ν, μ, λ, κ being constants.

For the system (1)-(3) we prove existence of a weak solution and also uniqueness in the case of one space dimension. For a simplified version of the equations, taking only mild coupling into considerations, the system can be controlled from the boundary and from sources as well in order to obtain a special structure of austenite-martensite lattice distribution in the material. We will demostrate a few numerical testexamples showing phase transitions and their control.